Termination of Initialized Two Variable Homogeneous Linear Loops

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Abstract—**In this paper, we consider the termination problems about uninitialized and initialized two variable homogeneous linear loops. Through analysing the properties of the boundaries of the set NT composed of all points on which a given uninitialized two variable homogeneous linear loop does not terminate, we reconstruct a group of simpler and complete quantified formulas to check whether a given uninitialized loop terminates. Furthermore, we present another group of quantified formulas to check the termination of initialized two variable homogeneous linear loops.**

Keywords-component; Program Verification;Loop Terminatio; QEPCAD; Quantifier Elimination

I. Introduction

In the contemporary era, various intelligent electronic devices have been integrated into our lives. Any fault happening in them may cause catastrophic consequences. How to automatically and accurately find errors in a program, is a grand challenge and has received much attention. As one of the bases for designing reliable software, the verification of loop's termination is a difficult problem and unable to always be proved [1].

Synthesizing ranking functions is the domain method for establishing termination. Through mapping each program state to a value from the well-founded domains by the ranking function and demonstrating each step in the loop reduces the value assigned by theranking function to guarantee there can be no infinite descending chain of elements of a well-founded domain, we can get the loop must eventually terminate. Hence, how to automatically generate such ranking function has been studied by [2-5].

However, the ranking functions of loops may not be easy to be discovered is the limitations of such method. In contrast to the method of using ranking function, Tiwari and Braverman in [6,7] prove the decidability of the termination problem for a special class of linear loops over the reals and the integers respectively. In [8], Xia and his colleagues further develop the work of [6] by symbolic computing. Following the work of [6,7], Xia and Zhang consider the termination problems of more general classes of programs and presents an algorithm to decide the termination of linear programs with nonlinear constraints over the reals in [9].

In addition, some other methods [10-13] are also available to analyse the termination of loop programs. For instance, through analysing finite difference trees, Bradley et al. in [10] present a sound but not complete method for proving termination of multipath loops with polynomial guards and assignments. In [11], Cook et al. present the first known automatic counterexample-guided abstraction refinement algorithm for termination proofs. Moreover, based on transitivity of ranking relations to prove program termination, an alternative algorithm that uses a light-weight check is given in [13].

In this paper, we first reconstruct a group of simpler and complete quantified formulas to determine the termination of uninitialized two variable homogeneous linear loop by analysing the boundaries of its NT. Then, we generalizes the method to decide the termination of the initialized one.

In [6], Tiwari presented the corresponding quantified formulas to check the termination of uninitialized two variable homogeneous linear loops. In [14], Li et al. find that a quantified formula which corresponds to a certain case of is neglected in Tiwari's results and give a group of complete quantified formulas to determine if the two variable homogeneous linear loop terminates. But, by analysis of the boundaries of NT, we can simplify the resulting quantifier formulas. This enables us to check whether uninitialized two variable homogeneous linear loops terminate completely and more simply.

In general, the initialized loop widely exists in practical code. In [15], Singh describes an order reduction technique and gives the reduction of the initialized termination problem to decide positivity of linear recurrences. Unlike [15], we determine if the initialized two variable homogeneous linear loops over $\Re$ terminate by checking the satisfiability of several quantified formulas. More importantly, given an initial input x^*, our method can construct a kind of loops which do not terminate on x^*. We will show this point by an example.

II. Preliminaries

This section introduces our loop abstraction and basic concepts.

Definition 1. The general structure of the uninitialized homogeneous linear loop over the tuple of program variables $x=(x_1,x_2)$ being the following form (denoted by L2):

$$L2: while(Bx>0)\{x:=Ax\}$$

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