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Computing the Singular Solution of Power Flow System

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Abstract. The purpose of this paper is to compute the singular solution of the nonlinear equations arising in power flow system. Based on the approximate null space of the Jacobian matrix, more equations are introduced to the origin system. Meanwhile, the Jacobian matrix of augmented equations at initial value is full rank, then the algorithm recovers quadratic convergence of Newton's iteration. The algorithm in this paper leads to higher accuracy of the singular solution and less iteration steps. In addition, two power flow systems are studied in this paper and the results show this new method has high accuracy and efficiency compared with traditional Newton iteration

Introduction

Determining the constant steady state solutions (i.e., equilibria) of models of interconnected power systems, known as the load (or power) flow problem, has become increasingly important, presenting challenges to theoretical as well as applied researchers. The load flow problem involves the calculation of line loading given the generation and demand levels for the normal balanced three-phase steady-state operating conditions of an electric power system. In general, load flow calculations are performed routinely for power system planning, and in connection with system operation and control. The solutions of the load flow can be used for the study of normal operating conditions, contingency analysis, system security assessment, optimal dispatching, characters of electricity net, as well as stability and bifurcations of (models of) power systems [1].

The load flow equations are modeled as a set of polynomial system

$$F(x) = 0. \tag{1}$$

where $F(x) = [f_1, f_2, \cdots, f_n]$ and $x = [x_1, x_2, \cdots, x_n]$.

There are many methods to solve the above equation, such as Groebner basis [2], triangular set [3] and resultant [4]. Actually, all of these methods mentioned above can obtain the exact solution of system (1), but they are restricted to small size systems because of the high complexity of the symbolic computation. As far as we know, there exists three kinds of numerical methods to solve nonlinear equations: Newton's iteration, homotopy continuation method and interval bisection method. Each kind of algorithm has its advantages and disadvantages. Given an appropriate initial point, the convergence of Newton's method is typical local quadratic. The solution of the power flow based on the Newton-Raphson's technique is first introduce in [5]. Since then, a huge variety of studies have been presented about the solution of the power flow problem, but how to choose the initial value is a difficult problem [6]. The homotopy algorithm and continuous Newton's method do not need to consider the initial value problem [7-9], and homotopy continuation method is a global convergence algorithm. But when the multiplicity of some solution are more than two, solving the system at this point is an ill-posed problem since the determinant of the Jacobian matrix equal to zero at exact solution. The convergence of these algorithms slows down when the approximate solution is closely to exact solution and we attains only a few correct digits of the exact zeros. Several techniques [10,11] have been proposed to get the singular solution of the power flow system.

This paper addresses the singular solution of the power flow system. In this goal, we modified the deflation algorithm introduced in [12,13] based on the approximate null space of the Jacobian matrix. The method is applied to compute the singular solution of 3 notes and 4 notes power flow system.

Mathematical Model of Load Flow

There are various kinds of representations of the (mathematical) models of power systems that can be chosen for the studies of specific problems. In this paper, we use the classical 'complex' polynomial models of power systems, see [1] for more details.

Consider an n+1 bus power system with r PQ notes, n-r PV notes, and a slack notes. The load flow equations for the general power system can be described as the following complex polynomial system:

For PV notes

$$\begin{cases} E_j \sum_{i=1}^{n+1} y_{j,i}^* E_i^* + E_j^* \sum_{i=1}^{n+1} y_{j,i} E_i - 2P_j = 0 \\ E_j E_j^* - V_j^2 = 0 \end{cases} \tag{2}$$

And the PQ notes

$$\begin{cases} E_j \sum_{i=1}^{n+1} y_{j,i}^* E_i^* - S_j = 0 \\ E_j^* \sum_{i=1}^{n+1} y_{j,i} E_i - S_j^* = 0 \end{cases} \tag{3}$$

where E_j is voltage of note j and it is a complex. Superscript $*$ means the conjugate, and $y_{j,i}$ is the admittance between node j and note i. P_j is input power of node j, V_j is the voltage amplitude of node j, S_j is input complex power of node j, N_g is the number of node PV and N is the number of nodes PQ.

Based on the formula (2, 3), we can convert the plow flow to a quadratic polynomial system. If each of the solution is not singular, then homotopy continuation method can help us get all of the roots. But otherwise, if there exists some singular solution, we introduced an efficient algorithm to get the singular solution.

Modified deflation for the singular solution of a nonlinear system

A symbolic deflation method was presented in [14], and further developed in [15-17]. For numerical case, Leykin et al. proposed a numerical deflation method based on the Jacobian matrix to compute the singular solution of a nonlinear system. There are also other algorithms based on different techniques, see [18-19] for more details. In this section, we give a modified deflation method based on the approximate right null space of the Jacobian matrix.

Suppose F is a polynomial system, and $\overline{x} = (\overline{x}_1, \cdots \overline{x}_n)$ is an approximate singular solution of F. Let $J_F(\overline{x})$ be the Jacobian matrix of F at point $\overline{x}$, and J_F the Jacobian matrix of F. We suppose that the approximate rank of $J_F(\overline{x})$ is k. Without loss of generality, we suppose that the first k column of $J_F(\overline{x})$ are linear independent. Let $\overline{y} = (\overline{y}_1, \cdots, \overline{y}_k, \overline{y}_{k+1}, \cdots, \overline{y}_n)$ be an approximate solution of matrix $J_F(\overline{x})$, and $(a_1, \cdots a_k)$ be a new variable set. The point $\overline{y}$ and k can be obtained through singular value decomposition of matrix $J_F(\overline{x})$.

Now, we construct a new polynomial system as follows:

$$G = [g_1, \cdots, g_n] = J_F \cdot X_1^T \quad (4)$$

where $X_1 = (a_1, \cdots, a_k, \overline{y}_{k+1}, \cdots, \overline{y}_n)$

It is easy to see that $\overline{X}_1 = (\overline{x}_1, \cdots, \overline{x}_n, \overline{y}_1, \cdots \overline{y}_k)$ is the approximate zero of the system $F_1 = [F, G]$. If matrix $J_{F_1}(\overline{X}_1)$ is rank deficit, we can construct a new system with the above method based on the system F_1 and point $\overline{X}_1$ until the Jacobian matrix of the augmented system at its corresponding approximate root is full rank. Suppose we obtain the full rank system denoted it by H, and its approximate root by $\overline{X}$, then the Newton iteration locally converges to a point which has a higher precision at a quadratic rate.

In the following, we give an algorithm to compute the singular solution of a nonlinear system.

Algorithm 1: RefineRoot

Input: A polynomial system F and approximate root $\overline{x}_0$ and a tolerance τ.

Output: An approximate zero $\overline{x}$ which has a higher precision than $\overline{x}_0$.

Step 1: Using singular value decomposition to determine the rank of the Jacobian matrix $J_F(\overline{x}_0)$.

Step 2: If matrix $J_F(\overline{x}_0)$ is full rank. Go to step 3

Otherwise, set s=0;

while s=0 do

constructing new system according to formula (4).

if the augmented system is full rank, break; and go to step 3; else let F be the augmented system, and $\overline{x}_0$ its approximate root.

End if;

End do

Step 3: $\overline{x}_{k+1} = \overline{x}_k - J_F(\overline{x}_k)^+ F(\overline{x})$.

In step 3, $J_F(\overline{x}_k)^+$ means the generalized inverse matrix of $J_F(\overline{x}_k)$, and we stop the Newton iteration algorithm when $\| x_{k+1} - x_k \| \leq 10^{-16}$ in this paper.

Numerical experiments

Algorithm RefineRoot has been implemented in Maple 15 named RefineRoot. In this section, deflation algorithm is applied to two power systems with 3 buses and 4 buses respectively.

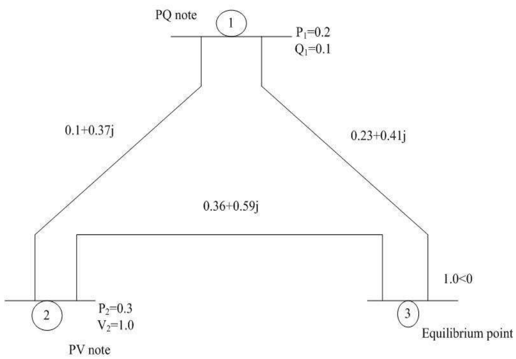

Figure 1: 3-Notes model system

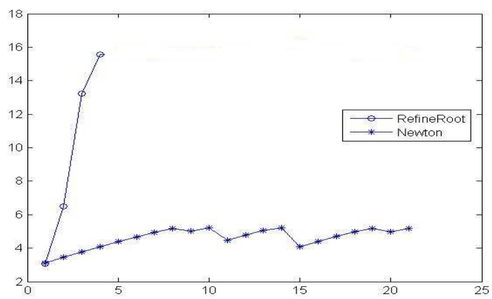

Figure 2: Comparisions between Newton's method and RefineRoot.

Example 1 : Fig 1 depicts a power system with 3 notes, according to formula (2,3), the power flow equation is

$$F=\begin{cases} .681 f_1 f_2 + 2.519 e_1 f_2 - 1.721 f_1^2 - 2.519 e_2 f_1 - 1.855 f_1 + 0.681 e_1 e_2 - \\ 1.721 e_1^2 + 1.041 e_1 - P_1 \\ 2.519 f_1 f_2 - 0.681 e_1 f_2 - 4.374 f_1^2 + 0.681 e_2 f_1 + 1.041 f + 2.519 e_1 e_2 - \\ 4.374 e_1^2 + 1.855 e_1 - 0.1 \\ 0.681 f_1 f_2 - 2.519 e_1 f_2 - 1.235 f_2 + 2.519 e_2 f_1 + 0.681 e_1 e_2 + 0.754 e_2 - 1.134 \\ f_2^2 + e_2^2 - 1 \end{cases}$$

When P_1 is in the vicinity at 1.47 and initial value $\overline{x}_0 = [-.417, -.433, .437, .902]$.

Figure 2 shows the difference between RefineRoot and Newton's method for the 3 notes flow load system. In this figure, ordinate axis denotes $-\lg(\| x_{k+1} - x_k \|)$ and horizontal axis represents the iteration steps. From the figure we can see that the convergence of standard Newton's method is at most linear since the singularity of the Jacobian matrix, but the convergence of algorithm RefineRoot is at least quadratic. This algorithm terminates after only four iterations.

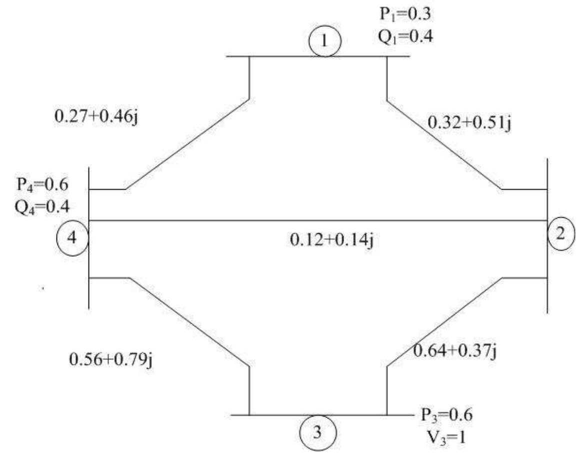

Figure 3: 4 Bus power system

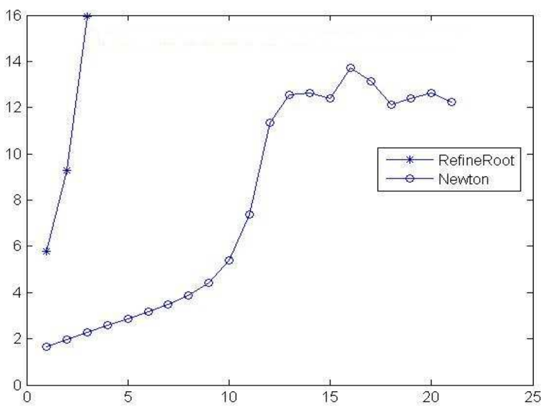

Figure 4: Comparisions between Newton's method and RefineRoot.

Example 2: Figure 3 describes a 4 buses power system, which has two PQ notes and one PV note, and the power flow equation is

$$F=\begin{cases} 1.832 a_1^2 - 0.949 a_1 a_2 - 1.617 a_1 b_2 + 1.617 b_1 a_2 - 0.949 b_1 b_2 - 0.883 a_1 + 1.407 b_1 \\ +1.832 b_1^2 - P; \\ 0.949 b_1 a_2 + 1.617 a_1 a_2 + 1.617 b_1 b_2 - 0.949 a_1 b_2 + 0.883 b_1 - 3.024 b_1^2 \\ +1.407 a_1 - 3.024 a_1^2 + 0.4; \\ 0.949 a_1 a_2 + 1.617 a_1 b_2 + 1.617 b_1 a_2 + 0.949 b_1 b_2 + 3.529 a_2 - 4.118 b_2 \\ -5.076 b_2^2 - 5.076 a_2^2 + 0.598 a_2 a_3 + 0.842 a_2 b_3 - 0.842 b_2 a_3 + 0.6 + 0.597 b_2 b_3; \\ 1.617 a_1 a_2 + 1.617 b_1 b_2 - 6.578 a_2^2 + 4.118 a_2 + 3.529 b_2 - 6.577 b_2^2 \\ -0.949 b_1 a_2 + 0.949 a_1 b_2 + 0.842 a_2 a_3 - 0.597 a_2 b_3 + 0.597 b_2 a_3 + 0.842 b_2 b_3 + 0.4; \\ 2.342 a_3 + 1.354 b_3 - 1.194 a_2 a_3 + 1.685 a_2 b_3 - 1.685 b_2 a_3 - 1.194 b_2 b_3 + 3.537 b_3^2 \\ +3.537 a_3^2 - 1.2; \\ a_3^2 + b_3^2 - 1; \end{cases}$$

In this example, the solution is very close to singualr. The difference between Newton's method and the algorithm RefineRoot for this special case is shown in Fig.4. From this figure, we see that the algorithm RefineRoot terminates through only 3 computations with accuracy of 16. But Newton's method only attains accuracy of 12 after 20 iterations.

Conclusion

A new deflation algorithm is developed for solving the singular solution of the power flow problem. First, augmented system is obtained by the initial value and the Jacobian matrix of power flow system, then using Newton's algorithm to solve the new system. This new method has been applied to 3 notes and 4 notes power flow system, the numerical result shows that our new method has less iteration step and higher accuracy.

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