Termination Analysis of Programs with Periodic Orbit on the Boundary

Jingmin Chen[1, a], Yi Li*[2, b], Guang Zhu[2, c], Chuancan Li[2, d], Fangjian Huang[3, e]

[1]School of Mathematics Sichuan University Chengdu, China.

[2]Chongqing Key Laboratory of Automated Reasoning and Cognition CIGIT, CAS.

[3]School of Energy Science and Engineering, University of Electronic Science and Technology of China Chengdu, China.

[a]jmchen808@126.com, [b]liyi@cigit.ac.cn, [c]zhuguang@cigit.ac.cn, [d]ChuancanLi@163.com, [e]huangfangjian@ustc.edu.cn

*Corresponding author

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Abstract. The termination problem of a class of simple while program: While (constraints) do {updates} end is proven to be decidable by computing periodic orbit of nonlinear updating function over the reals. The termination problem of such a program with open constraint domain which have periodic orbit on the boundary is also discussed and the corresponding algorithms are given.

Introduction

Plenty of computer software has been applied to people's work and entertainment in modern life. Because some bugs in software may cause catastrophic consequences, many software companies release bug repairing programs for their products frequently. The problem of program verification has been widely concerned by computer scientists (see [1-3]). Termination analysis as the essential problem in program verification is of great significance and extremely difficult.

In general, the classical approach for checking the termination of program is the synthesis of so-called ranking function which maps each program state to a value in a well-founded domain. The progress is that, by demonstrating that each step in the execution of program reduces the measure assigned by the ranking function, we can make sure such given program terminates. By constructing a ranking function of a given program, we make each process of program execution correspond to a chain of elements of the well-founded domain. Therefore, we conclude that the given program terminates. Namely, the existence of a ranking function of a given loop implies that such loop must terminate. Several methods about synthesizing ranking functions have been studied in [4,5,6,7,8,9].

In contrast to ranking function discovering, recently some algebraic approaches have been applied to program verification.

Let $\mathbb{R}$ denotes reals and $\mathbb{R}^n = \overbrace{\mathbb{R}\times\mathbb{R}\times\cdots\times\mathbb{R}}^{n}$. A simple while Program over $\mathbb{R}^n$ can be described specifically as follows:

$$\text{while} \quad (X \in \Omega) \text{ do } \{X := F(X)\} \text{ end.} \tag{1}$$

Where $X \in \mathbb{R}^n$, $\Omega \subset \mathbb{R}^n$ and $F : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous mapping. A. Tiwari[10] proved the decidability of a linear case of (1) as $\text{while}\,(BX > b)$ do $\{X := AX + c\}$ end. by real eigenvectors belong to positive eigenvalue of A, where A is an $n\times n$ matrix, B is an $m\times n$ matrix, x, b, and c are vectors. M. Braverman[11] discussed the termination of such a program over integers. To avoid errors caused by floating-point computation, Yang. L et al. [12, 13] further proposed a method to the termination of these programs by calculating symbolic conditions.

A nonlinear loop over reals can be described as P_1: while $(x \in \Omega)$ do $\{x := f(x)\}$ end. Where $\Omega \subset \mathbb{R}$, $f : \mathbb{R} \to \mathbb{R}$ is continuous. Some cases of P_1 were discussed by Yao [14] and an interesting result was given as follows:

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